International Journal of Engineering & Scientific Research
Vol.14 Issue 01, January 2026
ISSN: 2347-6532 Impact Factor: 8.076
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

STUDY OF IMPACT ON C-α-COMPACTNESS IN FUZZY TOPOLOGICAL SPACES

Vishwajeet Kumar Bihari
Research Scholar,Department of Mathematics
B.N.M.U Madhepura
&
Dr. Indrakant Jha
Department of Mathematics, R.M.College Saharsa

ABSTRACT

Fuzzy set theory provides us with a framework which is wider than that of classical set theory. Various mathematical structures, whose features emphasize the effects of ordered structure, may be developed on the theory. Fuzzy topology is one such branch, combining ordered structure with topological structure. We introduce the concept of fuzzy nearly C-α-compactness in fuzzy topological spaces and fuzzy bitopological spaces. For deriving some interesting properties, the properties of fuzzy almost continuous and fuzzy almost open functions are also discussed. we derive here characterisation of C-α-compactness in fuzzy setting. We derive properties related with the concept of fuzzy nearly C-α-compactness and some of the characterizations in fuzzy topological spaces. we derive the properties of fuzzy almost α-continuous, fuzzy almost α-open functions. We introduce the concept of fuzzy nearly C-α-compactness in fuzzy bitopological spaces. C. L. Chang introduced and developed the concept of fuzzy topological spaces based on the concept of a fuzzy set introduced by Zadeh.

Keywords:- fuzzy regular closed subset, bitopological spaces, fuzzy topological spaces, fuzzy bitopological spaces, fuzzy filter base,

Introduction

The concept of compactness to fuzzy topological spaces by different approaches. The significance results have been derived in the classical topology such by extended it to fuzzy topological spaces. The concept of nearly C-compactness in general topology has studied by P. L. Sharma. Let x denote fuzzy topological space, whereas by a fuzzy set in a non-void set X stand for a function λ from X to the unit closed interval [0, 1] (= I) i.e., $\lambda \in I^X$. By x_t it mean a fuzzy set whose value at the singleton support $\{x\}$ is t and 0 otherwise. For a fuzzy set λ in X, 1-λ denotes the fuzzy complement of λ and is defined by $(1 - \lambda)(x) = 1 - \lambda(x)$, for each x ∈ X. The notations Cl(λ) and Int(λ) will stand for the closure and interior respectively of the fuzzy set λ in the fts X. For any family $\{\lambda_i : i \in \Lambda\}$ of fuzzy sets in X, the union $\bigvee_{i\in\Lambda}\lambda_i$ and $\bigwedge_{i\in\Lambda}\lambda_i$ are defined as $(\bigvee_{i\in\Lambda}\lambda_i)(x) = \sup_{i\in\Lambda}\lambda_i$ and $(\bigwedge_{i\in\Lambda}\lambda_i)(x) = \inf_{i\in\Lambda}\lambda_i(x)$ for each x ∈ X, where Λ is an arbitrary index set. The characteristic function of a subset A of X is denoted by χi and is defined as

$$\mathcal{X}_A(x) = \begin{cases} 1 & if\ x \in A \\ 0 & if\ x \notin A \end{cases} \qquad \ldots(1.1)$$

Let $U = \{\lambda_\alpha\}_{\alpha\in\triangle}$ be a family of members from T. Then U is called a cover of X if $\bigvee_{\alpha\in\Lambda}\lambda_\alpha = 1$ and a subfamily of U having a similar property is called asubcover of U.

(i) Definition (Fuzzy topological space)

A fuzzy topological space (X, T) is said to be fuzzy compact if every cover of X by members of T has a finite subcover. Also, x_α denote a fuzzy point with a support x and value $\alpha (0 < \alpha \leq 1)$. For a fuzzy set λ in X, Let us write $x_\alpha \in \lambda$ provided $\alpha = \lambda(x)$.

(ii) Definition (Filter base)

We state basic definition and properties of fuzzy filter base on X is a non-empty collection F of fuzzy sets on X satisfying the following conditions:

(i) $0 \notin \mathfrak{I}$; where 0 stands for empty fuzzy set;

(ii) $\lambda_1, \lambda_2 \in \mathfrak{I} \Rightarrow \lambda_1 \wedge \lambda \in \mathfrak{I}$;

(iii) $\lambda < \mu \in \mathfrak{I} \Rightarrow \lambda \in \mathfrak{I}$.

(iii) Definition (Fuzzy regular)

A fuzzy set λ in a fts (X, T) is called

(i) fuzzy regular open set, if $Int(Cl(\lambda)) = \lambda$ and a fuzzy regular closed setif $Cl(Int(\lambda)) = \lambda$,

(ii) a fuzzy α-open set, if $\lambda \leq Int(Cl(Int(\lambda)))$ and a fuzzy α-closed set if$Cl(Int(Cl(\lambda))) \leq \lambda$.

(iv) Definition (α-regular)

A fuzzy topological space (X, T) is said to be fuzzy α-regular if for every fuzzy closed set λ and a point $x \notin \lambda$, there exists disjoint fuzzy α-open sets γ and μ such that $x \in \gamma$ and $\lambda < \mu$. A fuzzy set d is fuzzy a-regular open if $Int_\alpha(Cl_\alpha(\delta)) = \delta$.

(v) Definition(C-compact)

A topological space (X, T) is said to be nearly C-compact a if for any ordinary subset A of X, $A \neq X$ such that χ_A(the characteristic function of $A \subset X$) is a proper regular closed set and for each open cover of$\{\lambda_\alpha;\ \alpha \in \Delta\}$ of χ_A there exists a finite subfamily $\{\lambda_{\alpha 1}, \lambda_{\alpha 2}, \ldots\ldots, \lambda_{\alpha n}\}$ such that$\chi_A \subset \bigcup_{i=1}^{n} Cl\,(\lambda_{a_i})$.

(vi)Definition (Nearly compact)

A fuzzy topological space (X, T) is said to be fuzzy nearly C-compact if for any ordinary subset A of X, $A \neq X$ such that χ_A(the characteristic function of $A \subset X$) is a proper fuzzy regular closed set and for each fuzzy open cover of $\{\lambda_\alpha: \alpha \in \triangle\}$ of χ_A there exists a finite subfamily$\{\lambda_{\alpha 1}, \lambda_{\alpha 2}, \ldots, \lambda_{\alpha n}\}$ such that $\chi_A \leq \bigvee_{i=1}^{n} Cl\,(\lambda_{a_i})$.

International Journal of Engineering & Scientific Research
Vol.14 Issue 01, January 2026
ISSN: 2347-6532 Impact Factor: 8.076
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Fuzzy Nearly C-a-Compactness in Fuzzy Topological Spaces

(i) Definition (C-α compact)

A topological space is said to be nearly C-α-compact if given a regular closed set A and an α-open cover U = $\{O_i | i \in \wedge\}$ of A there exists a finite subfamily $\{O_i;\ i = 1, 2, \ldots, n\}$ of U such that $A \subset \cup_{i=1}^{n} Cl_{\alpha}(O_i)$

(ii) Definition

Let (X, T) be a fuzzy topological space. (X, T) is said to be fuzzy nearly C-α-compact if for any ordinary subset A of X, A ≠ X such that χ_A(the characteristic function of A ⊂ X) is a proper fuzzy regular closed set and for each fuzzy α-open cover of $\{\lambda_\alpha;\ \alpha \in \Delta\}$ of χ_A there exists a finite subfamily $\{\lambda_{\alpha 1}, \lambda_{\alpha 2}, \ldots\ldots, \lambda_{\alpha n}\}$ such that $\chi_A \le \vee_{i=1}^{n} Cl_{\alpha}(\lambda_{a_i})$. These definition shows that fuzzy compactness implies fuzzy nearly C-α-compactness. However, the converse is not true which is illustrated as follows.

Let X = {a, b}, T = $\{0, 1, f_n\}$ where f_n: X = {a, b} → [0, 1] is such that $f_n(X) = 1 - \frac{1}{n}$, $\forall x \in$ X. The only possible non-empty subsets of X are A_1= {a} and A_2= {b}. Further, since $ClInt\chi_{A1} = 0 \neq \chi_{A1}$ and $ClInt\chi_{A2} = 0 \neq \chi_{A2}$, it follows that χ_{A1} and χ_{A2} are not fuzzy regular closed. So vacously (X, T) is fuzzy nearly C-α-compact. Now we claim that (X, T) is not fuzzy compact. Indeed, of $\vee_{n=1}^{\infty} f_n = 1$ shows that $\{f_n\}_{n=1}^{\infty}$ is a fuzzy open coverof 1_X but for every infinite integer, say n_0, we have $\vee_{n=1}^{n_0} f_n < 1$ and therefore $\{f_n\}_{n=1}^{\infty}$ has no finite subcover for 1_X. That is (X, T) is not fuzzy compact.

International Journal of Engineering & Scientific Research
Vol.14 Issue 01, January 2026
ISSN: 2347-6532 Impact Factor: 8.076
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Fuzzy Almost α-Continuous and Fuzzy Almost α-Open Functions

(i) Definition

Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping $f : (X, T) \rightarrow (Y, S)$ is said to be fuzzy almost α-continuous if the inverse image of every fuzzy regular open (closed) set is fuzzy α-open (α-closed).

(ii) Definition

Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping $f : (X, T) \rightarrow (Y, S)$ is said to be fuzzy almost a-open (α-closed) if the image of every fuzzy regular open (closed) set is fuzzy α-open (α-closed).

Let X = {a, b, c}; Define T_1= {0, 1, λ} and T_2= {0, 1, μ}where λ(a) = 0, λ(b) $=\frac{2}{3}$, λ(c) $=\frac{1}{2}$and μ(a) = 1, μ(b) = 0, μ(c) = 0. Let $f : (X, T_1) \rightarrow (X, T_2)$ be the identity mapping. In (X, T_2) the only non-zero fuzzy regular open set is 1 and $f^{-1}(1) = 1$ shows that f is fuzzy almost α-continuous. Now let $g : (X, T_2) \rightarrow (X, T_1)$ be the identity mapping, the only non-zero fuzzy regular open set in (X, T_2) is 1 and $f(1) = 1$ implies that f is fuzzy almost α-open. We state some results derived by C. Chang A. Swaminathan.

Thereon

Let (X, T), (Y, S) and (Z, R) be fuzzy topological spaces. Let $f : X \rightarrow Y$ be a fuzzy almost α-open (almost α-closed) map of a space X onto a space Y and let $g : Y \rightarrow Z$. If $g\, of$ is a fuzzy almost α-continuous and fuzzy almost α-open map then g is fuzzy almost α-continuous.

Theorem

Let (X, T), (Y, S) and (Z, R) be fuzzy topological spaces. Assume that $f : X \rightarrow Y$ is fuzzy almost α-continuous and let $g : Y \rightarrow Z$ be a fuzzy almost α-continuous and fuzzy almost α-open map. Then $g \circ f$ is fuzzy almost α-continuous.

Theorem

Let (X, T), (Y, S) and (Z, R) be fuzzy topological spaces. Let $f : X \to Y$ and $g : Y \to Z$ and suppose that ($g \circ f$) is a fuzzy almost α-open (fuzzy almost α-closed) map. If f is fuzzy almost α-continuous and fuzzy almost α-open surjection, then g o f is a fuzzy almost α-open (almostα-closed) map.

Lemma

Let (X, T) and (Y, S) be fuzzy topological spaces. Let $f : X \to Y$ be any fuzzy almost α-open map. Given any $\lambda \in I^Y$ and any fuzzy regular closed set μ containing $f^{-1}(\lambda)$, there exists a fuzzy α-closed set $\theta \geq \lambda$ such that $f^{-1}(\theta) \leq \mu$.

Lemma

Let (X, T) and (Y, S) be fuzzy topological spaces. Let $f : X \to Y$ be a fuzzy almost α-continuous and fuzzy almost α-open map and let λ be any fuzzy α-closed subset of Y . Then

(i) $Cl_\alpha(f^{-1}(Int_\alpha(\lambda))) = f^{-1}(Cl_\alpha(Int_\alpha \lambda))$

(ii) If λ is a fuzzy α-regular closed subset, then $Cl_\alpha(f^{-1}(Int_\alpha\lambda)) = f^{-1}(\lambda)$

Theorem

Let (X, T) and (Y, S) be fuzzy topological spaces. Let $f : X \to Y$ be a fuzzy almost α-continuous and fuzzy almost α-open map. If μis a fuzzy α-open set in Y, then

(i) $Int_\alpha Cl_\alpha(f^{-1}(Int_\alpha(Cl_\alpha\mu))) = Int_\alpha[Cl_\alpha(f^{-1}(Cl_\alpha\mu))] = Int_\alpha(f^{-1}(cl_\alpha\mu))))$

(ii) $f^{-1}(Int_\alpha(Cl_\alpha\mu)) = Int_\alpha(f^{-1}(Cl_\alpha\mu))$

(iii) If μ is a fuzzy α-regular open set in Y, then $Int_\alpha f^{-1}(f^{-1}{}_\alpha(Cl_\alpha(\mu)) = f^{-1}(\mu)$.

International Journal of Engineering & Scientific Research
Vol.14 Issue 01, January 2026
ISSN: 2347-6532 Impact Factor: 8.076
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Theorem

The image of a fuzzy nearly C-α-compact space under a fuzzy almost α-continuous and fuzzy almost α-open mapping is fuzzy nearly C-α-compact.

Proof

Let f : (X, T) → (Y, S) be a fuzzy almost α-continuous and fuzzy almost α-open mapping from a fuzzy nearly C-α-compact X onto Y. It is to be proved that Y is also fuzzy nearly C-α-compact. Let A be any subset of Y such that χ_A is fuzzy regular closed in Y. Let U = $\{\lambda_i\}_{i\in\Delta}$be a fuzzy regularα-open cover of χ_A in Y. $f^{-1}(\chi_A)$ is a fuzzy regular closed subset of X and$\{f^{-1}(\lambda_i)_{i\in\Delta}$is a fuzzy regular α-open cover of $f^{-1}(\chi)$ in X. Since X is fuzzy nearly C-α-compact, there exists a finite subfamily $\{f^{-1}(\lambda_i)$; I = 1, 2, . . ., n$\}$such that

$$f^{-1}(\mathcal{X}_A)\leq\bigvee_{i=1}^{n} cl_\alpha\{f^{-1}(\lambda_i)\}\leq\bigvee_{i=1}^{n}\{f^{-1}(cl_\alpha(\lambda_i))\} \qquad \text{... (1.2)}$$

That is $\mathcal{X}_A = \vee_{i=1}^{n}\{cl_\alpha(\lambda_i)\}$. This proves that Y is fuzzy nearly C-α-compact.

Fuzzy C-compactness provides a more flexible generalization of standard fuzzy compactness, allowing researchers to study topological spaces where exact binary membership is relaxed.

References:-

1. Misra, K. B., & Weber, G. G. (1990) Use of fuzzy set theory for level-I studies in probabilistic risk assessment. Fuzzy Sets and Systems, 37, 139–160.
2. Puri, M. L., & Ralescu, D. A. (1986) Fuzzy random variables. Journal of Mathematical Analysis and Applications, 114, 409–422.
3. N. Cagman, S. Karatas and S. Enginoglu, (2011) Soft topology, Comput. Math. Appl. 62, 351-358.
4. Gil, M. A., Corral, N., & Gil, P. (1985) The fuzzy decision problem: An approach to the point estimation problem with fuzzy information. European Journal of Operational Research, 22, 26–34.
5. Dubois, D., & Prade, H. (1979) Fuzzy real algebra: Some results. Fuzzy Sets and Systems, 2, 327–348.
6. Pervin, W. J. (1967) Connectedness in bitopoligcal space, Duke Math. J V 34 (367-392)
7. Naudi, J. N (1994) Bitopological near compactness Bull. Al. Math. Soc 86 (147-156)